Seminario PRISMA

Friday, February 19, 2021 5:06 PM

Sketch of proof

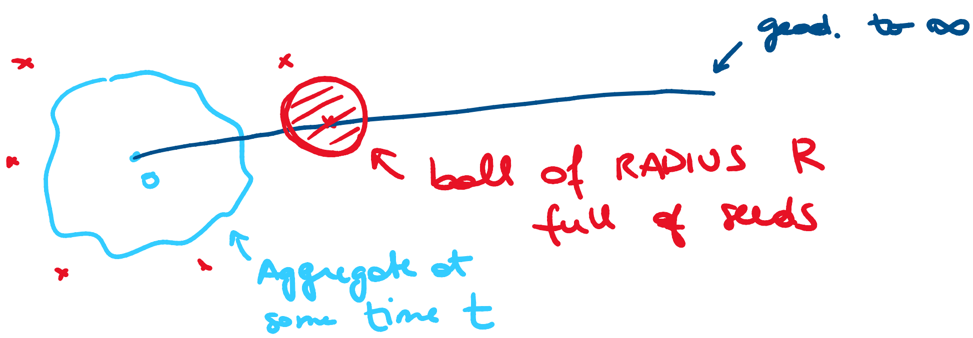
(*) G infinite, hyperbolic, vertex-transitive

$\Longrightarrow$ $\forall \mu \in (0,1)$, $\forall \lambda > 0$

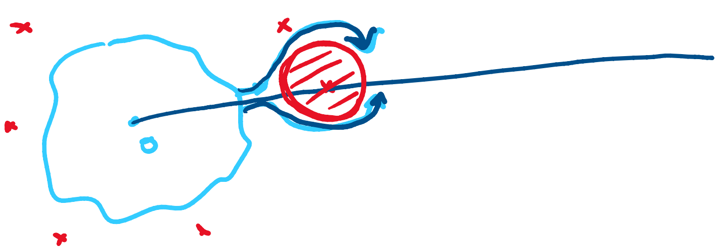
$\mathbb{P}_\mu(\mathrm{FPP}_\lambda \text{ survives}) = 1.$

Idea:

Fix a large constant $\underline{\underline{R}}$



(it occurs w.p.p.)



When ball is activated, to stop FPP_λ we need to SURROUND the ball

Aggregate at time T

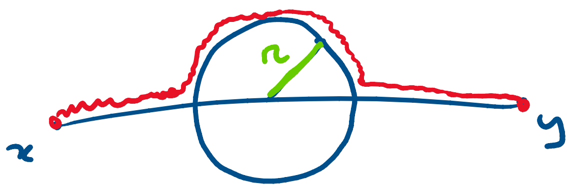
1st CRUCIAL FACT: FPP spreads

of time T

1st CRUCIAL FACT: FPP spreads LINEARLY in time (whp: $B(o, c\cdot T) \subseteq A_T \subseteq B(o, C\cdot T)$)

2nd CRUCIAL FACT: (Result by Gromov):

δ-HYPERBOLICITY $\Rightarrow$ exponential detours!

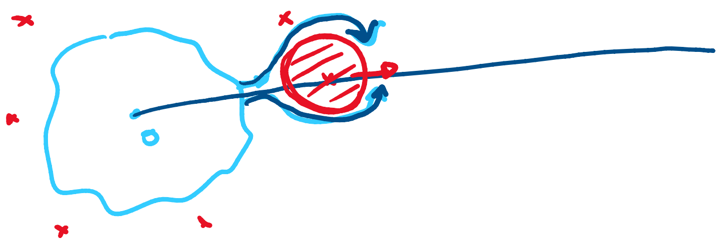


If $r \gg \delta$,

$x, y \notin$ Ball

$\Rightarrow$ RED PATH has

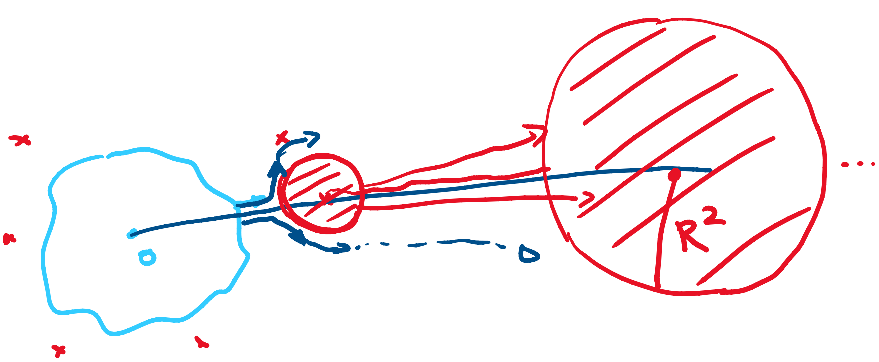
Length $\geq \delta \cdot 2^{r/\delta}$



⊛ Time needed by FPP_λ to proceed $\approx R$

⊛ Time needed by FPP_1 to proceed $\approx 2^{R/\delta}$.

(R is LARGE).



After time $\sim 2^{R/\delta}$

FPP_1 very delayed

& FPP_λ occupies

& FPP_λ occupies
a larger ball (radius $\sim R^2$)
which will delay FPP_1
even more.

– And so on –

Each such attempt is successful wpp $\Rightarrow$ eventually
we will see
a success. $\square$